Research Spotlight: National Autonomous University of Mexico: **Fractal geometry and fractional calculus used to model gas flow and gas diffusion in unconventional reservoirs**

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The National Autonomous University of Mexico (UNAM), located in Mexico City, is one of the largest universities by enrollment, is consistently ranked among the top universities in Latin America, and is an active member of the Mexico InterPore Chapter. The Earth Science Engineering Division (DICT) is part of the university's Faculty of Engineering, and includes the Geology, Geophysics, Mining, and the Petroleum Engineering Departments. Part of this department's mission is to conduct research on fluid flow modeling and enhanced oil recovery in both conventional and unconventional (shale) petroleum reservoirs. In particular, this department develops pressure and flow rate transient analytical models that are designed to dynamically characterize complex naturally fractured reservoirs, including shales. Currently our group has 15 members, including faculty, and undergraduate, master's, and doctoral students.

Problem

Most reservoirs have naturally existing fractures, and some of them, like shales, are stimulated by hydraulic fracturing because these reservoirs have very low permeability. For this reason, shales are usually called unconventional reservoirs.

One of the research areas of our group is to model traditional oil reservoirs and unconventional shale reservoirs in order to optimize oil production. Stimulating the reservoir by hydraulic fracturing causes originally isolated natural reservoir microfractures to be interconnected in the vicinity of the hydraulic fracture, increasing flow rates. This stimulation process to the microfracture system does not occur uniformly due to the anisotropy of the reservoir stress state, and results in two fractured domains: a stimulated three-dimensional zone and an unstimulated region where the original porosities and permeabilities are present. The presence of microfractures, nano and micropores, and organic matter, results in a highly heterogeneous triple-porosity domain with a velocity field that has long-range spatial and temporal dependencies.

In shale gas reservoirs, three forms of storage can be identified: free gas compressed within the porous matrix of the rock or within its natural fractures, gas adsorbed to organic matter and mineral surfaces within the natural fractures and the porous matrix, and gas dissolved in the kerogen (primary organic compound of oil shale) (Ambrose, et al. 2010). Because of the vast orders of magnitude (from the nano scale to the macro scale), it is important to consider different scale-dependent transport mechanisms. Pore sizes can be as small as the mean free path of the gas particles. In these conditions the Knudsen number shows rarefaction of the gas, so that it is important to consider the Knudsen diffusion effect (nano scale) in contrast to the viscous flow permeability used in Darcy's law (macro scale) (Karniadakis, et al. 2006; Ziarani and Aguilera 2012). On the larger end, in the hydraulic fractures, the pore sizes can be large enough where Darcy's law may be valid.

Modeling Approach

Horizontal well configurations with multiple hydraulic fractures have made production from unconventional shale-type reservoirs economically feasible. Our goal is to realistically evaluate the petroleum reserve production that can be obtained from horizontal wells with hydraulic fracturing as well as to design the number of hydraulic fractures and horizontal wells that will optimize production in shale.

For both conventional reservoirs and shales we use fractal geometry to describe the distribution of natural fractures, which is one way to consider anomalous diffusion, and use machine learning to develop models that predict gas flow production. Numerical simulations are obtained using Compute Unified Device Architecture (CUDA). Here we present a research approach for numerically modeling the gas flow in shales through hydraulically fractured horizontal wells.

Currently, commercial well-performance modeling approaches are limited by their inability to capture the petrophysical complexities of the natural fracture system and all the flow mechanisms, including the presence of non-Darcy flow, that contribute to production. Traditional approaches (Brown, et al. 2011) usually represent the stimulated reservoir volume (SRV) as the region between the hydraulic fractures and extends from the horizontal well to the tips of the hydraulic fractures. The same description is used in the proposed approach but using the fractal distribution of natural fractures in the SRV instead of using a uniform distribution of natural fractures.

The classical double-porosity approach of modeling a fractured porous media (Barenblatt, Zheltov and Kochina 1960, and Warren and Root 1963) consists of two media: the matrix, which has low permeability and high storage, and the natural fracture network, which has a relatively high permeability and low storage. The matrix serves as a source for the natural fracture network, where the fluids are ultimately transported to the wellbore. Both the matrix and the natural fractures are characterized by averaged petrophysical properties that are evenly distributed throughout the system. Usually in both media, fluid flow is described by Darcy's Law, which establishes flow as proportional to an instantaneous and local pressure gradient. This type of idealization is appropriate for naturally fractured reservoirs where the fractures form a well-connected, uniformly distributed network, and where the matrix is relatively homogeneous and reasonably permeable. But for unconventional reservoirs, these assumptions can result in an overestimated flow rate producing and an overly optimistic oil production.

One modeling approach our group is exploring is on using a disordered and scale-dependent distribution of partially connected fractures along with a distribution of petrophysical properties, using fractal geometry (O'Shaugnessy and Procaccia 1985; Barker 1988; Chang and Yortsos 1990). This framework challenges the concept of constant matrix block size usually used to model naturally fractured reservoirs (Barenblatt, Zheltov and Kochina 1960, and Warren and Root 1963). With the fractal approach the matrix block size can be distributed as a function of scale (Fuentes-Cruz and Valkó 2015; Valdes-Perez and Blasingame 2018) or, more generally, a matrix shape factor distributed as a function of scale can be considered.

Another focus area is modeling anomalous diffusion (a term commonly used to describe flow of fluid in shale) resulting from non-Darcy flow observed in the matrix and natural fracture network. Flow time dependencies are modeled by generalizing diffusion equations to fractional diffusion equations, i.e. incorporating fractional derivatives (Camacho-Velázquez, et al. 2008). One result is that gas production rates are less in fractal-fractional systems when compared to the classical double-porosity approach of modeling naturally fractured systems with the same matrix permeability. One proposed model is presented by Raghavan (2011), where the anomalous diffusion is incorporated via a fractional-order time derivative in the continuity equation and the flow equation.

There have been several approaches taken on modeling flow in horizontal wells with hydraulic fractures accounting for anomalous diffusion approach has had some (Ozcan, et al. 2014; Wei, et al. 2015; Albinali and Ozkan 2016; Holy and Ozkan 2016; Gu, et al. 2017; Liu, et al. 2018), where, due to the linearity of the equations, some geometric restrictions have been made that allow analytical or semi-analytical solutions to be obtained in these references. Combining the mechanisms considered in these analytical and semi-analytical solutions with other flow mechanisms such as adsorption/desorption that are scale dependent have barely begun to be studied.

We model the petrophysical properties as being distributed due to the geometric characteristics of the hydraulic fracture network. The geometric characteristics of the hydraulic fracture are usually determined by the stress state of the rock before and after each fracture stage (see e.g. Xu and Wong 2010). Under this assumption, and considering that the in-situ stress state is kept constant after the hydraulic fracture treatment, we use the geometric characteristics of the hydraulic fracture to construct an anisotropic permeability tensor. The fracture network becomes more densely interconnected the closer it is to the fracture in any direction, so we distribute the petrophysical properties of the fracture system as a power law that covers the 3D space as a Matryoshka Doll, where the layers resemble the shape of the hydraulic fracture.

Results

Figure 1 shows a diagram of the simulated region and the coordinate system.

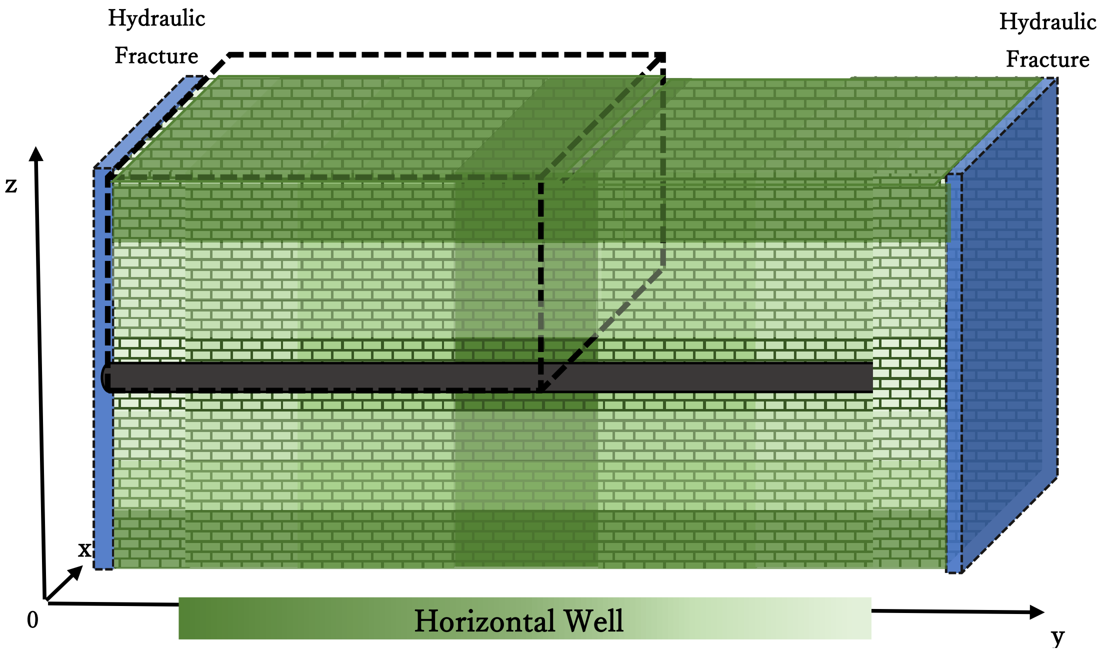

Figure 1. Numerically simulated region and coordinate system.

Figure 2 shows the spatial distribution of the permeability tensor main components from Figure 1 simulated data (Albinali and Ozkan 2016) for a horizontal well with hydraulic fractures in a fractured medium in 3 dimensions. The 3 set of graphs show the (eigenvalues of the permeability tensor, or the x-component, y-component, and z-component of the permeability tensor (left to right). The 4 inserted graphs are: upper left corner is parallel to the XZ plane and adjacent to the hydraulic fracture plane corresponding to the naturally fractured medium; in the upper right is a component of the permeability tensor parallel to the YZ plane at x=0, where the horizontal well is intersected by the hydraulic fracture, see Figure 1; in the lower right the XY plane with z=0, which is orthogonal to the hydraulic fracture; and in the lower left corner the plane corresponding to the center of the hydraulic fracture. The permeability spectrum spans from 10 millidarcy (red) to 0.0001 millidarcy (blue).

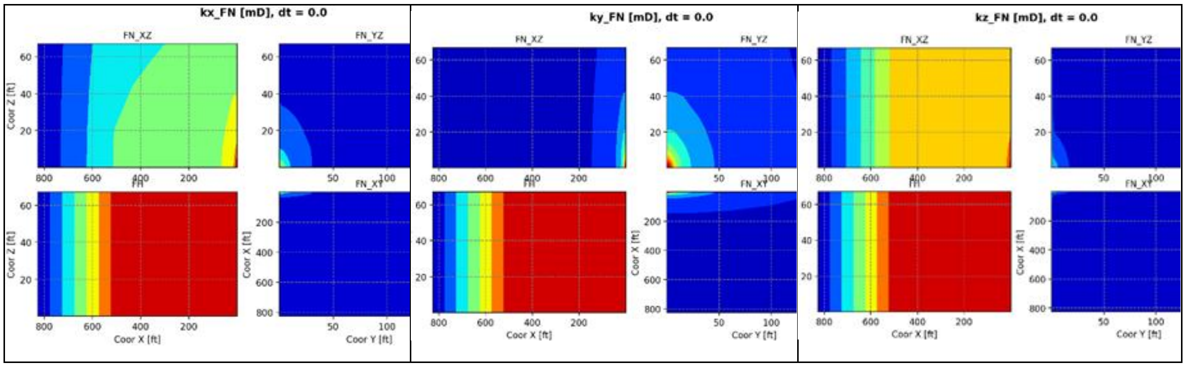

Figure 2. Spatial distribution of permeability tensor components.

Figure 3 shows the differences in the spatial distribution of porosity. On the left side the porosity distribution is made by means of the geometry of the hydraulic fracture, using the simulated data of Figure 1, considering that the fracture network becomes more densely interconnected the closer it is to the fracture in any direction, we distribute the petrophysical properties of the fracture system as a power law that covers the 3D space as a Matryoshka Doll. On the right side the porosity distribution is made using the following equations (O'Shaugnessy and Procaccia 1985; Chang and Yortsos 1990), for the permeability tensor:

$$k_{NFj} = k_{HF}\left(\frac{j}{\frac{w_F}{2}}\right)^{d_{fj}-\theta_j-1}$$

where d_{fj} is the fractal dimension in the direction $j = x, y, z$, and θ_j is the connectivity index in the direction j, and for the porosity:

$$\phi = \frac{\phi_{FH}}{3}\left\{\left(\frac{x}{\frac{w_F}{2}}\right)^{d_{fx}-1}\left(\frac{y}{\frac{w_F}{2}}\right)^{d_{fy}-1}\left(\frac{z}{\frac{w_F}{2}}\right)^{d_{fz}-1}\right\}$$

where d_f *is* the fractal dimension, which is between 0 (where there are no fractures) and 1 (for a uniformly distributed fractures), and θ *is the* connectivity index, which is between 0 (for a fully interconnected fracture network) and 1 (for an isolated fractures). For the right side of Figure 3, the fractal parameters are given by: $d_{fx} = 1.976$, $d_{fy} = 1.95$, $d_{fz} = 1.997$, $\theta_x = 1$, $\theta_y = 0.1$ and $\theta_z = 1$.

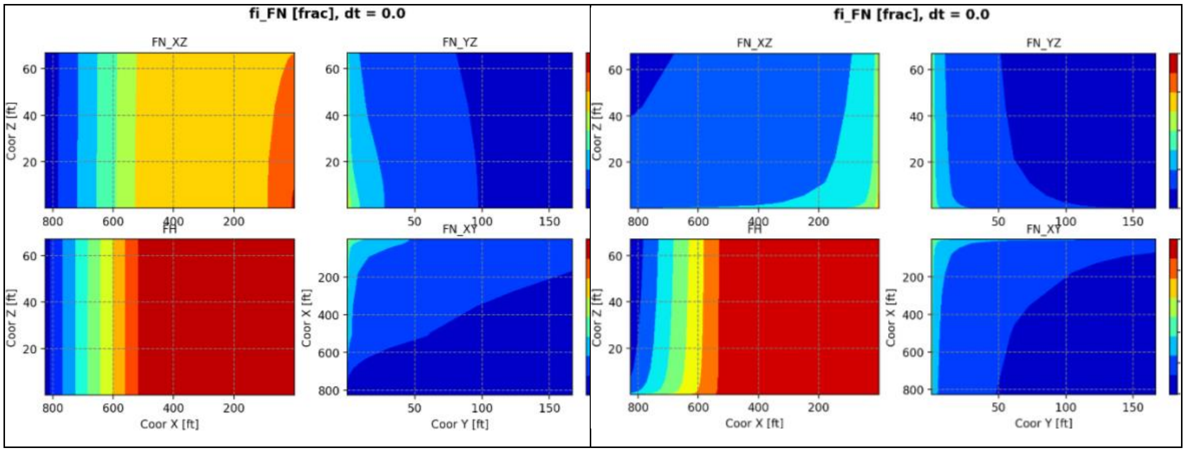

Figure 3. Spatial porosity distribution.

Figure 4 compares the simulation of an artificially generated reservoir modeled by assuming the petrophysical properties are homogenous (blue), fractal geometry (red) using a distribution of the petrophysical properties of the fracture system as a power law that covers the 3D space as a Matryoshka Doll, and fractal formulas (green). It is observed that the homogeneous case overestimates the production at long-times after 0.1 days. The two fractal cases, denoted by red and green curves, do not present an identical behavior after 0.1 days but they are close; however, these differences are very noticeable at short times. The previous fractal equations yield properties of permeability and porosity where stimulation penetrates deeper into the volume compared to the geometric distribution given by the Matryoshka Doll approach, so that at short times the flow rate obtained with the use of the fractal equations is greater than the rate obtained with the geometrical approach.

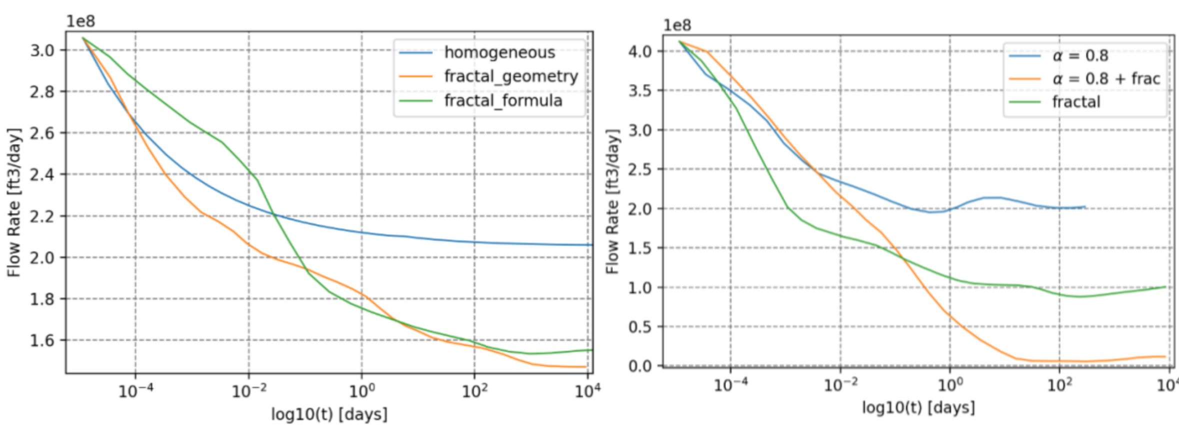

Figure 4. Flow rate comparison between homogeneous case and two views of 3D fractal distribution.

Figure 5. Combined effect of anomalous diffusion and distribution of fractal petrophysical properties.

Some numerical simulations were performed combining the effects of anomalous diffusion, represented by using non-integer derivative orders and/or the 3D distribution of fractal porosity and permeability of the fracture network These simulations were carried out assuming no-flow boundary conditions, with only one horizontal well and the volume between two hydraulic fractures.

The results are presented in Figure 5 where a comparison of the cases are (1) a fractional derivative of order α =0.8 (blue line) with no fractal petrophysical property; (2) a fractal distribution of petrophysical properties with the following fractal parameters d_{fx}=1.88, d_{fy}=1.97, d_{fz}=1.91, θ_x=0.9, θ_y=0.15 and θ_z=0.9, and no fractional dispersion (represented by the green line); and (3) a combined effect, i.e. fractional-fractal approach, whose results are presented as the red line in this figure. A large difference in the production can be observed between these three sub-diffusive processes. Thus, it is convenient to use field data to determine which of these proposed approaches is more appropriate to use.

We have developed a numerical model that incorporates a fractal distribution of petrophysical properties of the fracture network, taking into account a non-uniformly distributed and non-fully interconnected fracture network, and anomalous diffusion by incorporating fractional derivatives that accounts fpr the presence of non-Darcy flow. In future work, we are planning to include additional validation of these simulations by comparing the simulation results with some analytical solutions available in the technical literature and possibly also by comparing with experimental data.

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